

MTH6109 Combinatorics

Coursework 1

Hand in your solution (to the starred question only) to the RED BOX on the GROUND FLOOR of the Mathematical Sciences Building before 5pm on Monday 12 October.

1. Show that a set $X = \{x_1, x_2, \dots, x_n\}$ of n distinct elements can be made into a sequence $(y_1, y_2, \dots, y_n)$ of length n in exactly $n!$ ways.

2.* Making use of the formula

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad 0 \leq k \leq n$$

established in the course, show the following.

a) $\binom{n}{k} = \binom{n}{n-k}, \quad 0 \leq k \leq n;$

b) $k \binom{n}{k} = n \binom{n-1}{k-1}, \quad 1 \leq k \leq n;$

c) $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}, \quad 1 \leq k \leq n+1.$

3. Give alternative proofs for the formulae of Question 2, by showing that the left-hand and right-hand sides count the elements of one and the same set, respectively of equipotent sets.

4. Show that $\binom{n}{0} = \binom{n}{n} = 1$, and that $\binom{n}{k} = 0$ for $k > n$.